

UPPSC-AE

2025

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination
Assistant Engineer

Mechanical Engineering

Design of Machine Elements

Well Illustrated **Theory** *with*
Solved Examples and Practice Questions



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Design of Machine Elements

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Design Against Fluctuating Load

1.1 Introduction to Fatigue Loads

Fatigue loads are those whose magnitude and direction or both magnitude and direction change w.r.t time and same load repeatedly applied.

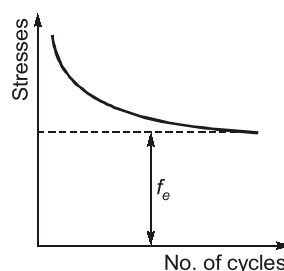
- Machine is a combination of resisting bodies which have some relative motion which is used to transform one form of energy into mechanical energy. **Ex.** : Aircraft, Automobile, car etc.
- Design of machine element is an integral part of mechanical design in which designer create devices to satisfy human needs.
- A design engineer always prefers ductile material compared to brittle material because ductile material gives indication in the form of yielding before fracture but in the brittle material immediate fracture occurs, i.e., no yielding.

1.1.1 Fatigue

- Fatigue is a phenomenon associated with variable loading or cyclic stressing just like animals and humans get fatigue when specific task (applying specific stress) is repeatedly performed.
In this manner components subjected to variable loading get fatigue which leads to their premature failure is known as fatigue failure.
- Most mechanical components experience variable loading due to change in the magnitude or direction of applied load.
- **Worst case** of fatigue loading is fully-reversible load.

1.1.2 Fatigue Failure

- When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue**. The fatigue failure may occur even without any prior indications. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads, the number of load reversals and shape of the components and irregularities present in it.



- If the stress is kept below a certain value as shown by dotted line, the material will not fail whatever may be the number of cycles. This stress as represented by dotted line is known as **endurance or fatigue limit (f_e)**. It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^6 cycles).
- The term **endurance limit** is used for reversed bending only; while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material.

1.2 Types of Fatigue Stresses

1.2.1 Fluctuating Fatigue Stress

- The stresses which vary from a minimum value to a maximum value of the same nature (i.e. tensile or compressive) are called fluctuating fatigue stresses.

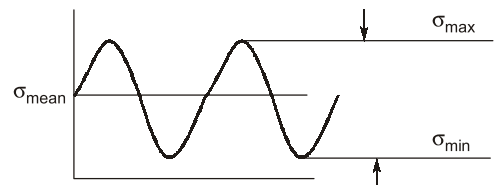


Figure: Fluctuating stress cycle

1.2.2 Repeated Fatigue Stress

- Stress variation is such that the minimum stress is zero to a maximum stress value and mean and amplitude stress have the same value for repeated loading.

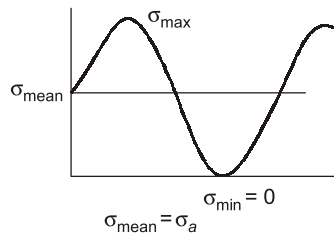


Figure: Repeated stress cycle

1.2.3 Cyclic Stress/Completely Reversed Fatigue Stress

- The stresses which vary from one value of compressive to the same value of tensile or vice versa, are known as completely reversed or cyclic fatigue stresses.

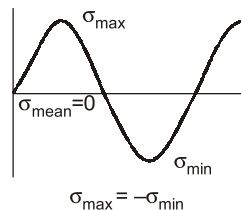


Figure: Completely reversed or cyclic stress

1.2.4 Alternating Fatigue Stress

- The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called alternating fatigue stresses.

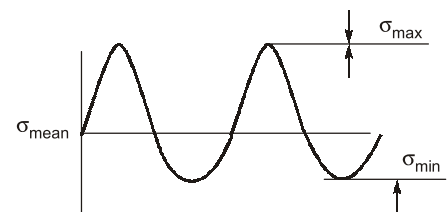


Figure: Alternating Stress

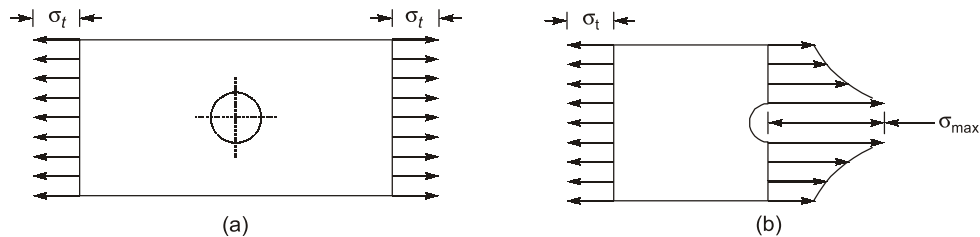
1.3 Some Values of Stress

• Means stress,	$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$
• Stress amplitude,	$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$
• Stress range,	$\sigma_r = \sigma_{\max} - \sigma_{\min}$
• Stress ratio,	$R = \frac{\sigma_{\max}}{\sigma_{\min}}$
• Amplitude ratio,	$A = \frac{\sigma_a}{\sigma_m}$

here σ_{\max} = Maximum stress value during complete cycle
 σ_{\min} = Minimum stress value during complete cycle

1.4 Stress Concentration/Stress Concentration Factor

- **Stress concentration** is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section.



1.4.1 Stress Concentration Factor (k_t)

$$k_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations for minimum cross-section}}$$

or

$$k_t = \frac{\sigma_{\max.}}{\sigma_0} = \frac{\tau_{\max.}}{\tau_0}$$

1.4.2 Fatigue Stress Concentration Factors (k_f)

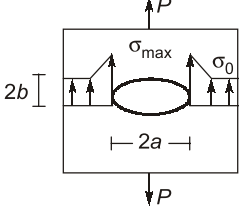
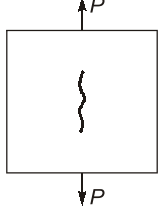
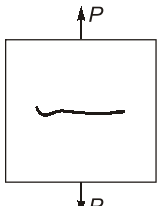
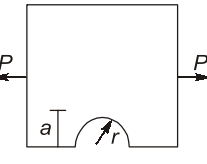
$$k_f = \frac{\text{Maximum stress in notched specimen}}{\text{Stress in notch free specimen}}$$

$$k_f = \frac{\text{Endurance limit of a notch free specimen}}{\text{Endurance limit of a notched specimen}}$$

1.4.3 Stress Concentration Due to Hole

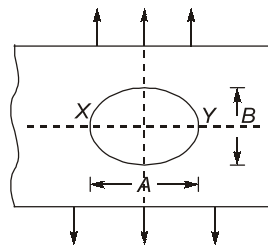
- The stress at the joints away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole.

$$\sigma_{\max} = \sigma \left(1 + \frac{2a}{b} \right)$$

 $k_t = \frac{\sigma_{\max}}{\sigma_0} = \left(1 + \frac{2a}{b} \right)$ <p>For circular hole $a = b$ $k_t = 3$</p> <p>(a) Elliptical Hole</p>	 $a = 0$ $k_t = \frac{\sigma_{\max}}{\sigma_0} = 1$ $\sigma_{\max} = \sigma_0$ <p>(b) Crack parallel to load</p>	 $b = 0$ $k_t = \frac{\sigma_{\max}}{\sigma_0} = \infty$ $\sigma_{\max} = \infty$ <p>(c) Crack perpendicular to load</p>	 <p>a - depth of notch r - radius of notch</p> $k_t = \frac{\sigma_{\max}}{\sigma_0} = 1 + \frac{2a}{r}$ <p>(d) Circular Notch</p>
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Example - 1.1 A loaded semi-infinite flat plate is having an elliptical hole ($A/B = 2$) in the middle as shown in the figure. Find the stress concentration factor at either X or Y is



Solution :

As we know,

Stress concentration factor,

$$k_T = 1 + \frac{2A}{B}$$

$$k_T = 1 + 2 \times 2 = 5$$



Example - 1.2 A cold rolled steel shaft is design on the basis of maximum shear stress theory (MSST). The principal stresses induced at its critical section are 60 MPa and -60 MPa respectively. If the yield stress for the shaft material is 360 MPa, find the factor of safety of the design.

Solution :

Given: $\sigma_1 = 60$ MPa; $\sigma_2 = -60$ MPa

So,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{60 + 60}{2} = 60 \text{ MPa}$$

According to MSST, yield stress in shear = S_{se}
 $S_{se} = 0.5S_{yt} = 0.5 \times 360 = 180 \text{ MPa}$

$$\Rightarrow \tau_{\text{allowable}} = \frac{S_{se}}{\text{FOS}}$$

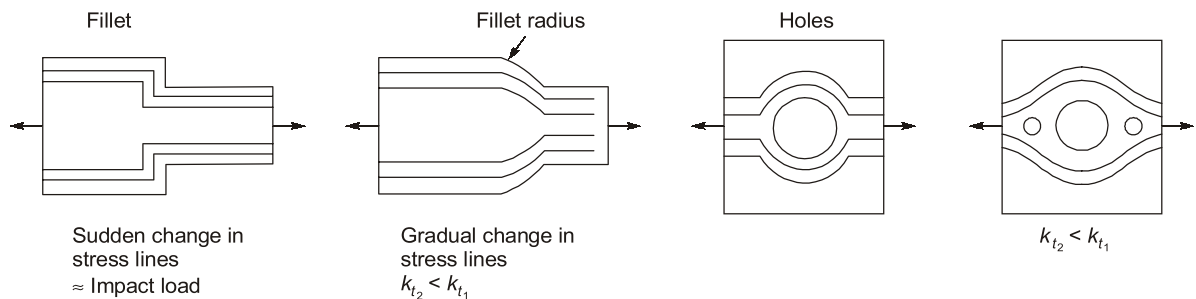
$$\Rightarrow \text{F.O.S.} = \frac{S_{se}}{\tau_{\text{allowable}}} = \frac{180}{60} = 3$$

1.5 Causes of Stress Concentration

- Geometric discontinuities like cracks, sharp corners, holes, cross-sectional changes etc.
- Discontinuity in applied loads.
- Material discontinuities occurred during manufacturing.

1.5.1 Methods to Avoid Stress Concentration

- Avoid sharp edges.
- If a crack is present then drill a large hole at the end of the crack.
- Make the stress intensity uniform if already a notch is there then make more notches for uniform strength.



1.5.2 Notch Sensitivity

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising in fatigue loading. The notch sensitivity factor q is defined as

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

Since

σ_0 = Nominal stress as obtained by elementary equations

\therefore

Actual stress = $k_f \sigma_0$

Theoretical stress = $k_t \sigma_0$

Increase of actual stress over nominal stress = $(k_f \sigma_0 - \sigma_0)$

Increase of theoretical stress over nominal stress = $(k_t \sigma_0 - \sigma_0)$.

k_f - fatigue stress concentration factor

Notch sensitivity,

$$q = \frac{k_f \sigma_0 - \sigma_0}{k_t \sigma_0 - \sigma_0} = \frac{k_f - 1}{k_t - 1}$$

$$k_f = 1 + q(k_t - 1)$$

**NOTE**

- (i) K_t depends on shape, size of discontinuity and type of load condition and its orientation.
- (ii) K_t is independent of material behaviour.
- (iii) K_f depends on shape, size of orientation and type of loading condition and material of component.
- (iv) Notch sensitive index (q) depends on material.
- (v) If $q = 0 \Rightarrow K_f = 1 \Rightarrow$ material is insensitive to stress concentration or notch.
- (vi) If $q = 1 \Rightarrow K_f = K_t \Rightarrow$ material is highly sensitive to notch.
- (vii) For worst design, take $q = 1$.

1.5.3 How to Improve Fatigue Strength

- By residual compressive stresses. Because fatigue failure is always a tensile failure. Therefore residual compressive stresses will counter it. This is done by the process called **shot peening**.
- By providing fillets, we can change its macrostructure thus we can basically increase its mechanical property that is fatigue strength.
- **Hammering:** The hammering process has been used to improve the fatigue resistance of the components.
- **Cold rolling:** Cold rolling gives excellent surfinish and increases material strength due to work hardening.
- **Burnishing:** In this metal is plastically deformed and convert the uneven surface of workpiece at normal temperature to smooth surface so as to change the surface structure, mechanical properties, shape and size. It improves the fatigue and wear strength of the workpiece.

1.5.4 Revised Endurance Limit (σ'_e)

Endurance limit is not a true property of material it can be changed and depend on lots of factors such as:

- | | |
|---|--|
| 1. Size factor, (k_{size}) | 2. Load factor, (k_{load}) |
| 3. Surface finish factor, (k_{sf}) | 4. Reliability factor, ($k_{reliability}$) |
| 5. Temperature factor, k_{temp} | 6. Modification factor, k_{modi} |
| 7. Miscellaneous, ($k_{miscellaneous}$) | |

If designer considers effect of all these factors, then Endurance strength value can be determined by given equation

$$\sigma'_e = k_{sf} k_{temp} k_{load} k_{size} k_{reliability} k_{modi} k_{miscellaneous} \sigma_e$$

1.6 Gerber Line/Soderberg Line/Goodman Line

1. Gerber Line:	A parabolic curve joining σ_e on the ordinate to σ_{ut} on the abscissa is called the Gerber line.
2. Soderberg Line	A straight line joining σ_e on the ordinate to σ_{yt} on the abscissa is called the Soderberg line.
3. Goodman Line	A straight line joining σ_e on the ordinate to σ_{ut} on the abscissa is called the Goodman line.

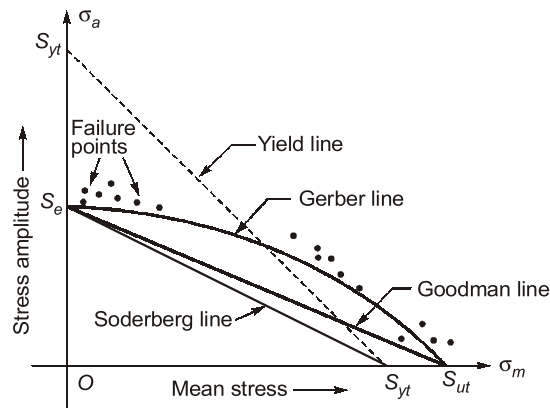


Figure: Gerber, Goodman and Soderberg lines

1.6.1 Gerber Method

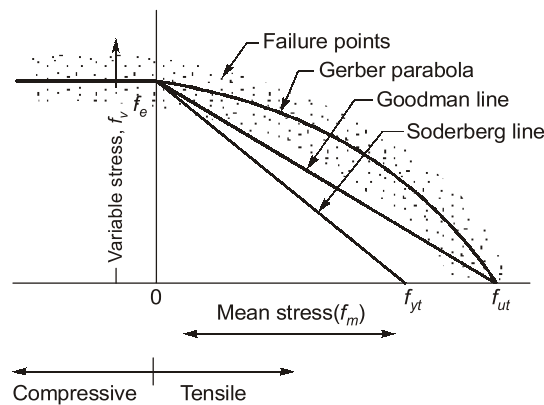


Figure: Gerber lines

f_e = Fatigue strength corresponding to the case of complete reversal ($f_m = 0$)

f_{ut} = Static ultimate strength corresponding to $f_v = 0$

- Generally, the test data for ductile material fall closer to Gerber Parabola, but because of scatter in the test points, a straight line relationship (i.e. Goodman line and soderberg line) is usually preferred. According to **Gerber**,

$$\frac{1}{F.S.} = \left(\frac{f_m}{f_u} \right)^2 F.S. + \frac{f_v}{f_e}$$

where, $F.S.$ = Factor of safety

- Considering fatigue stress concentration factor (k_f)

$$\frac{1}{F.S.} = \left(\frac{f_m}{f_u} \right)^2 F.S. + \frac{f_v \cdot k_f}{f_e}$$

1.6.2 Goodman Method

- A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials. Line AB connecting f_e and f_u is called Goodman's failure stress line. If a suitable factor of safety ($F.S.$) is applied to endurance limit and ultimate strength, a safe stress line CD may be drawn parallel to the line AB .

$$\frac{1}{F.S.} = \frac{f_m}{f_u} + \frac{f_v}{f_e}$$

Considering the load, surface finish and size factor.

$$\frac{1}{F.S.} = \frac{f_m}{f_u} + \frac{f_v k_f}{f_e \cdot K_{sur} \cdot K_{sz}}$$

- Here we have assumed the same factor of safety (F.S.) for the ultimate tensile strength (f_u) and endurance limit (f_e). In case the factor of safety relating to both these stresses is different then

$$\frac{f_v}{f_e / (F.S.)_e} = 1 - \frac{f_m}{f_u / (F.S.)_u}$$

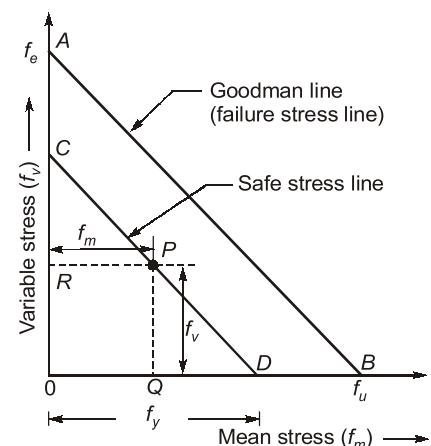
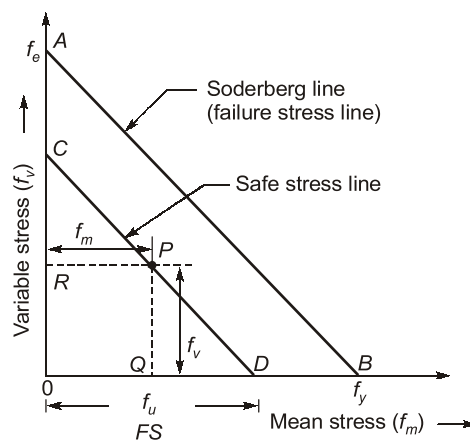


Figure: Goodman lines

1.6.3 Soderberg Method

- A straight line connecting the endurance limit (f_e) and the yield strength (f_y) is a soderberg line. This line is used when the design is based on yield strength.



- If a suitable factor of safety (F.S.) is applied to the endurance limit and yield strength, a safe stress line CD may be drawn parallel to the line AB.

$$\frac{1}{F.S.} = \frac{f_m}{f_y} + \frac{f_v \cdot k_f}{f_e}$$

Considering the load factor, surface finish factor and size factor the relation is

$$\frac{1}{F.S.} = \frac{f_m}{f_y} + \frac{f_v \cdot k_f}{f_{eb} \cdot K_{sur} \cdot K_{sz}}$$

NOTE: The Soderberg method is particularly used for ductile materials. For a reversed shear loading:

$$\frac{1}{F.S.} = \frac{f_{ms}}{f_{ys}} + \frac{f_{vs} \cdot k_{fs}}{f_{es} \cdot K_{sur} \cdot K_{sz}}$$



Example - 1.4 A cylindrical shaft is subjected to an alternating stress of 100 MPa. Fatigue strength to sustain 1000 cycles is 490 MPa. If the corrected endurance strength is 70 MPa, What will be the estimated shaft life?

Solution:

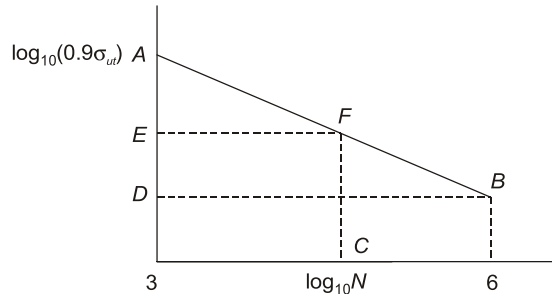
It is a finite life problem. The line AB is the failure line. Where $A\{3, \log_{10}(0.9\sigma_{ut})\}$ But here it will be $A\{3, \log_{10}(490)\}$ and $B\{6, \log_{10}(\sigma_e)\}$ Here it is $B\{6, \log_{10}(70)\}$

Therefore $F\{\log_{10} N, \log_{10}(100)\}$ we have to find N

$$\frac{EF}{AE} = \frac{DB}{AD}$$

$$\text{or } \frac{\log_{10} N - 3}{\log_{10} 490 - \log_{10} 100} = \frac{6 - 3}{\log_{10} 490 - \log_{10} 70}$$

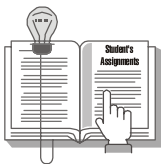
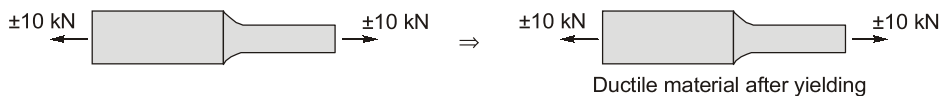
$$\text{or } N = 281914 \text{ cycles.}$$



Example - 1.5 Why stress concentration in a machine component of ductile materials is not so harmful as it is in brittle material?

Solution :

Stress concentration in a machine component of ductile materials results in, local yielding may distribute stress concentration.



Student's Assignment

Q.1 Notch sensitivity (q)

- (a) $\frac{k_f - 1}{k_t - 1}$ (b) $\frac{1 - k_f}{k_t - 1}$
(c) $\frac{1 - k_f}{1 - k_t}$ (d) Both (a) and (c)

Q.2 Equation of Goodman line is given by

- (a) $\frac{\sigma_m}{s_{yt}} + \frac{\sigma_a}{s_e} = 1$ (b) $\frac{s_{yt}}{\sigma_m} + \frac{\sigma_a}{s_e} = 1$
(c) $\frac{\sigma_m}{s_{ut}} + \frac{\sigma_a}{s_e} = 1$ (d) $\frac{\sigma_m}{s_{ut}} + \frac{s_e}{\sigma_a} = 1$

Q.3 Ratio of increase of actual stress over nominal stress to increase of theoretical stress over nominal stress is called

- (a) Endurance limit
(b) Fatigue strength
(c) Mean fluctuating stress
(d) Notch sensitivity

Q.4 Stress concentration factors are used for component made of brittle material subjected to

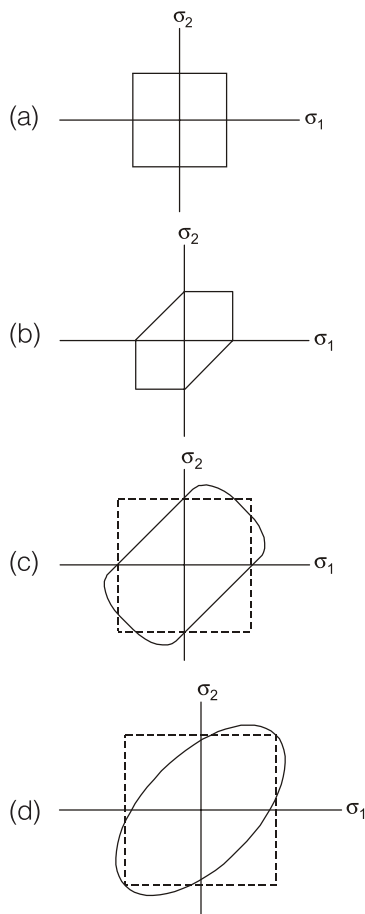
- (a) Static load (b) Fluctuating load
(c) Both (a) & (b) (d) None of these

2. used for rigid fastening of two rods.
 3. rods to be connected are subjected to tensile or compressive stresses along their axes.
 4. not suitable for joining members under rotation.
- (a) 1, 2 and 4 (b) 1, 3 and 4
(c) 1 and 2 (d) 1, 2, 3 and 4

- Q.29** The cotter is uniform in thickness but tapered in width on one side. The normal value of this taper is
- (a) 1 : 8 (b) 1 : 15
(c) 1 : 25 (d) 1 : 40

- Q.30** A localized compressive stress at the area of contact between two members is known as
- (a) Tensile stress (b) Bending stress
(c) Bearing stress (d) Shear stress

- Q.31** Which one of the following graph represents von-mises yield criterion



- Q.32** The piston rod and the cross head in a steam engine are usually connected by means of
- (a) Cotter joint (b) Knuckle joint
(c) Ball joint (d) Universal joint

ANSWER KEY
**STUDENT'S
ASSIGNMENT**

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (d) | 4. (c) | 5. (d) |
| 6. (b) | 7. (c) | 8. (c) | 9. (c) | 10. (b) |
| 11. (b) | 12. (d) | 13. (c) | 14. (d) | 15. (a) |
| 16. (b) | 17. (b) | 18. (c) | 19. (d) | 20. (b) |
| 21. (c) | 22. (b) | 23. (b) | 24. (b) | 25. (a) |
| 26. (b) | 27. (b) | 28. (d) | 29. (c) | 30. (c) |
| 31. (c) | 32. (a) | | | |

HINTS & SOLUTIONS
**STUDENT'S
ASSIGNMENT**

- 1. (d)**

$$\text{Notch sensitivity } (q) = \frac{K_f - 1}{K_t - 1} = \frac{1 - K_f}{1 - K_t}$$

- 2. (c)**

Equation of Goodman line

$$\frac{\sigma_m}{\frac{S_{yt}}{N_2}} + \frac{\sigma_v}{\frac{S_e}{N_1}} = 1$$

$$\text{For } N_1 = N_2 = 1 \Rightarrow \frac{\sigma_m}{S_{yt}} + \frac{\sigma_v}{S_e} = 1$$

- 3. (d)**

Notch sensitivity

$$= \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

- 4. (c)**

For brittle material, stress concentration factors are taken into account in both static and fluctuating load.